

Constrained CHOMP using Dual Projected Newton Method

Sanjiban Choudhury, Sebastian Scherer

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Robotics Institute
Carnegie Mellon University
Pittsburgh, Pennsylvania 15213

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Abstract

CHOMP is a popular trajectory optimization algorithm that uses covariant gradient techniques to produce high quality solutions. In its original formulation, it solves an unconstrained sequentially quadratic problem with extensions for handling equality constraints. In this paper we present an approach to solve sequentially quadratic problem with linear inequality constraints. We present a dual projected newton method to efficiently solve this problem. The proposed method alternates between primal and dual updates thus leading to faster convergence than solving a constrained quadratic program at each iteration.

1 Unconstrained CHOMP

Let $\xi \in \mathbb{R}^{n \times d}$ be a waypoint trajectory with n intermediate waypoints of dimension d . Let $\mathcal{U}[\xi]$ be a cost functional and $\bar{\nabla}\mathcal{U}[\xi]$ be the gradient. Let A be a metric tensor in which the norm between trajectories is measured. Let η be a regularization coefficient.

CHOMP[2] defines an unconstrained sequential quadratic problem at each iteration i as follows

$$\xi_{i+1} = \arg \min_{\xi} \mathcal{U}[\xi_i] + (\xi - \xi_i)^T \bar{\nabla}\mathcal{U}[\xi_i] + \frac{\eta_i}{2} \|\xi - \xi_i\|_A \quad (1)$$

The update step to this is closed form as follows

$$\xi_{i+1} = \xi_i - \frac{1}{\eta_i} A^{-1} \bar{\nabla}\mathcal{U}[\xi_i] \quad (2)$$

2 CHOMP with linear inequality constraints

Let $C\xi \leq d$ be a linear inequality constraint on the trajectory. The modified optimization problem at each iteration i is as follows

$$\begin{aligned} \xi_{i+1} = \arg \min_{\xi} \mathcal{U}[\xi_i] + (\xi - \xi_i)^T \bar{\nabla}\mathcal{U}[\xi_i] + \frac{\eta_i}{2} \|\xi - \xi_i\|_A \\ \text{s.t } C\xi \leq d \end{aligned} \quad (3)$$

The optimization problem in (3) is a quadratic programming problem.

3 Dual Projected Newton Method

3.1 Dual Problem

Let u be the Lagrange multipliers. The Lagrangian is as follows

$$\mathcal{L}[\xi, u] = (\xi - \xi_i)^T \bar{\nabla}\mathcal{U}[\xi_i] + \frac{\eta_i}{2} \|\xi - \xi_i\|_A + u^T (C\xi - d) \quad (4)$$

The Lagrange dual function is as follows

$$\begin{aligned} g[u] &= \min_{\xi} \mathcal{L}[\xi, u] \\ &= -\frac{1}{2\eta_i} (\bar{\nabla}\mathcal{U}[\xi_i] - C^T u)^T A^{-1} (\bar{\nabla}\mathcal{U}[\xi_i] + C^T u) + \\ &\quad u^T \left(C\xi_i - d - \frac{1}{\eta_i} CA^{-1} (\bar{\nabla}\mathcal{U}[\xi_i] + C^T u) \right) \end{aligned} \quad (5)$$

Hence the dual optimization problem at every iteration is as follows

$$\begin{aligned}
u_i &= \arg \max_u g[u] && \text{s.t } u \geq 0 \\
&= \arg \max_u -\frac{1}{2\eta_i} (\bar{\nabla}\mathcal{U}[\xi_i] - C^T u)^T A^{-1} (\bar{\nabla}\mathcal{U}[\xi_i] + C^T u) + \\
&\quad u^T \left(C\xi_i - d - \frac{1}{\eta_i} C A^{-1} (\bar{\nabla}\mathcal{U}[\xi_i] + C^T u) \right) && \text{s.t } u \geq 0 \\
&= \arg \max_u -\frac{1}{2\eta_i} u^T C A^{-1} C^T u + u^T \left(C\xi_i - d - \frac{1}{\eta_i} C A^{-1} \bar{\nabla}\mathcal{U}[\xi_i] \right) && \text{s.t } u \geq 0 \\
&= \arg \min_u \frac{1}{2\eta_i} u^T C A^{-1} C^T u - u^T \left(C\xi_i - d - \frac{1}{\eta_i} C A^{-1} \bar{\nabla}\mathcal{U}[\xi_i] \right) && \text{s.t } u \geq 0
\end{aligned} \tag{6}$$

The primal solution can be recovered from the dual as follows

$$\xi_{i+1} = \xi_i - \frac{1}{\eta_i} A^{-1} \bar{\nabla}\mathcal{U}[\xi_i] - \frac{1}{\eta_i} A^{-1} C^T u_i \tag{7}$$

Note that the difference in the update rule (7) from (1) is the addition of a term due to lagrange multipliers.

3.2 Projected Newton Method

The problem defined in (6) is a quadratic box constrained optimization problem. For cases where the Hessian $C A^{-1} C^T$ is non-singular, such problems can be solved using projected Newton methods [1].

Consider the following box constrained problem.

$$\min_u \mathcal{G}[u] \quad \text{s.t } u \geq 0 \tag{8}$$

The method starts with an initial point u_0 , a small constant $\varepsilon > 0$ and performs the following steps at iteration i

1. Define a binding set $\mathcal{B}_i = \{k : u_i^k \leq \varepsilon, [\bar{\nabla}\mathcal{G}[u_i]]_k > 0\}$, where u_i^k is the k^{th} variable of vector u_i , and $[\bar{\nabla}\mathcal{G}[u_i]]_k$ is the k^{th} variable of the gradient $\bar{\nabla}\mathcal{G}[u_i]$. These are variables close to the boundary and moving them further would decrease the criterion.
2. Define the free set $\mathcal{F}_i = \{1, \dots, n\} \setminus \mathcal{B}_i$.
3. Define the principal submatrix of the inverse Hessian along the free variables $S_i = [\bar{\nabla}^2 \mathcal{G}[u_i]]_{\mathcal{F}_i}$
4. A projected Newton step is taken along the free variables only

$$u_{i+1} = P_{\geq 0} \left(u_i - \alpha_i \begin{bmatrix} S_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} [\bar{\nabla}\mathcal{G}[u_i]]_{\mathcal{F}_i} \\ [\bar{\nabla}\mathcal{G}[u_i]]_{\mathcal{B}_i} \end{bmatrix} \right) \tag{9}$$

where $P_{\geq 0}$ projects each variable to positive half, α_i is the step size.

3.3 Algorithm

We propose a dual projected newton method to solve the constrained sequential quadratic problem (3). We start of with an initial primal trajectory ξ_0 , initial dual variables u_0 and at iteration i perform the following steps

1. Perform one iteration of projected Newton

$$u_{i+1} = P_{\geq 0} \left(u_i - \alpha_i \begin{bmatrix} S_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} [\bar{\nabla} \mathcal{G} [u_i]]_{\mathcal{F}_i} \\ [\bar{\nabla} \mathcal{G} [u_i]]_{\mathcal{B}_i} \end{bmatrix} \right) \quad (10)$$

$$\text{where } \mathcal{G} [u] = \frac{1}{2\eta_i} u^T C A^{-1} C^T u - u^T \left(C \xi_i - d - \frac{1}{\eta_i} C A^{-1} \bar{\nabla} \mathcal{U} [\xi_i] \right)$$

2. Perform one primal update

$$\xi_{i+1} = \xi_i - \frac{1}{\eta_i} A^{-1} \bar{\nabla} \mathcal{U} [\xi_i] - \frac{1}{\eta_i} A^{-1} C^T u_{i+1} \quad (11)$$

The basic idea behind the algorithm is that instead of ensuring the constraints are satisfied at each interim stage of CHOMP, a balance is maintained between objective minimization and constraints satisfaction. Since the original problem is non-convex, there is always a possibility that enforcing hard constraints initially might result in a poor local minimum. Instead in our algorithm, the cost minimization moves the trajectory away from obstacles first. As the trajectory approaches the local minimum, the quadratic approximations of the cost functions remain invariant thus allowing the dual ascent to converge and result in a feasible trajectory.

From a computation standpoint, this is also efficient as the time complexity of the update step is well defined (as opposed to solving the QP at each iteration depends on the conditioning of the problem).

4 Results

To illustrate the algorithm we will present some canonical results of 2D trajectory optimization with different constraints.

The cost and runtimes for all experiment and algorithm are shown in Table. 1.

Table 1: Performance of CHOMP variants for 3 experiments

Experiment	Unconstrained		Iterative QP		Dual Proj Newton	
	Time(s)	Cost	Time(s)	Cost	Time(s)	Cost
No constraints	0.016	1.019	-	-	-	-
Trajectory constraint	-	-	0.775	2.838	0.031	2.894
Goal constraint	-	-	0.421	0.862	0.0728	0.8158

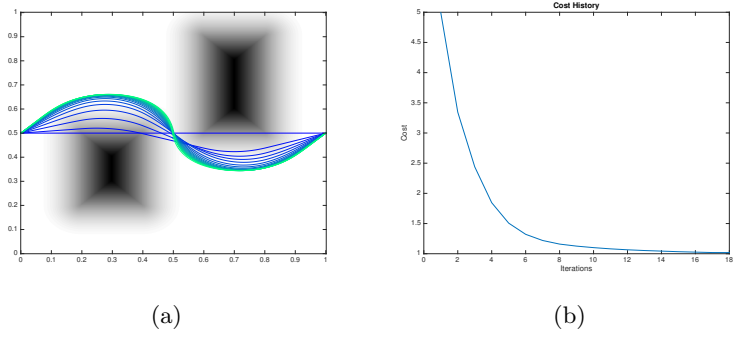


Figure 1: Unconstrained CHOMP (a) The cost map is visualized in gray scale along with all intermediate trajectories in the optimization (blue to green). (b) The cost at each iteration

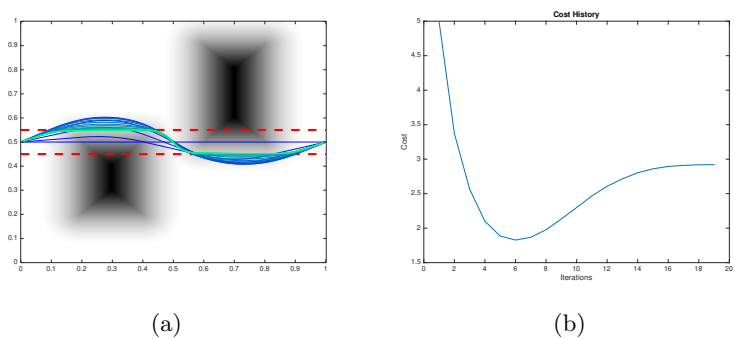
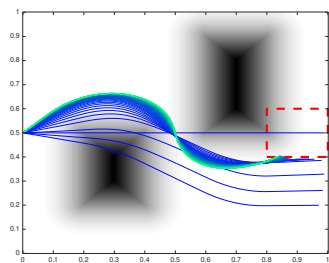


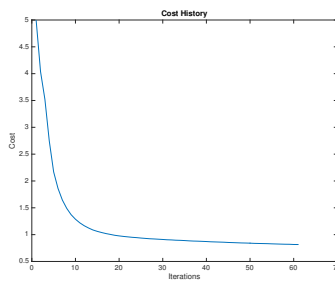
Figure 2: Constrained CHOMP with trajectory constraints (a) The cost map is visualized in gray scale along with all intermediate trajectories in the optimization (blue to green). The red dashed lines shows the trajectory constraints (b) The cost at each iteration. Note the cost increases at the end due to the dual ascent taking precedence.

5 Code

The MATLAB code for CHOMP is open source and available at https://bitbucket.org/castacks/matlab_chomp/overview



(a)



(b)

Figure 3: Constrained CHOMP with goal constraints (a) The cost map is visualized in gray scale along with all intermediate trajectories in the optimization (blue to green). The red dashed lines shows the goal constraints (b) The cost at each iteration.

References

- [1] Dongmin Kim, Suvrit Sra, and Inderjit S Dhillon. Tackling box-constrained optimization via a new projected quasi-newton approach. *SIAM Journal on Scientific Computing*, 32(6):3548–3563, 2010.
- [2] Matt Zucker, Nathan Ratliff, Anca D Dragan, Mihail Pivtoraiko, Matthew Klingensmith, Christopher M Dellin, J Andrew Bagnell, and Siddhartha S Srinivasa. Chomp: Covariant hamiltonian optimization for motion planning. *The International Journal of Robotics Research*, 32(9-10):1164–1193, 2013.